

Historic, archived document

Do not assume content reflects current scientific knowledge, policies, or practices.

477.7
F-7644n
cop. 2



INTERMOUNTAIN FOREST & RANGE EXPERIMENT STATION
507 - 25th STREET, OGDEN, UTAH 84401

FEMR 103

USDA FOREST SERVICE RESEARCH NOTE INT-190

STEM SURFACE AREA EQUATIONS FOR FOUR TREE SPECIES OF NEW MEXICO AND ARIZONA

David W. Hann and Robert K. McKinney, Jr.¹

ABSTRACT

Presents surface area equations for ponderosa pine ("yellow" and "blackjack" pine both separately and in combination), Douglas-fir, and a combination of Mexican white pine and white fir. The basic data came from three National Forests in Arizona and New Mexico, and were analyzed using weighted least squares regression techniques. Also presented are equations for predicting surface area of forked trees.

OXFORD: 536

KEYWORDS: *stem, surface area, weighted least squares regression, unforked trees, forked trees, analysis of covariance, residuals*

The total stem surface area of a tree is the area of the bole surface inside bark, excluding the end areas, from the ground to the tip of the tree. It is equivalent to the area of a sheet of paper wrapped around a form which approximates a paraboloid or cone. The use of surface area, when combined with cubic-foot volume and length (tree height), to predict product recovery from trees and logs has been described by Grosenbaugh (1954), Davis and others (1962), and Bruce (1970). Loxen (1943) and Husch (1963) suggested total stem surface area per acre as a measure of growing stock for estimating stand growth. Stand surface area inside bark is equivalent to area of the lateral meristem, where volume growth occurs.

The lack of surface area equations which could be used in product recovery studies in the Southwest stimulated the research reported here. This paper describes methods used to develop surface area equations as a function of total tree height and d.b.h. for four species (one with two vigor types) or species groups.

¹The authors are Assistant Resource Analyst, Intermountain Station, and graduate student at Utah State University, Logan. Mr. McKinney is now a Computer Programs Analyst, Weyerhaeuser Company, Tacoma, Washington.

SOURCE AND NATURE OF DATA

The data were collected by field crews from the Division of Timber Management, Southwestern Region, USDA Forest Service, located in Albuquerque, New Mexico.

Species and Forests were:

Douglas-fir (*Pseudotsuga menziesii* var. *glauca* [Mirb.] Franco) from the Lincoln and Tonto National Forests.

Mexican white pine (*Pinus flexilis* var. *reflexa* Engelm.) from the Lincoln National Forest.

White fir (*Abies concolor* [Gord. and Glend.]) from the Lincoln National Forest.

"Blackjack pine" (*Pinus ponderosa* Laws.) from the Lincoln, Coconino, and Tonto National Forests.

"Yellow pine" (*Pinus ponderosa* Laws.) from the Lincoln, Coconino, and Tonto National Forests.

Foresters in the Southwest separate "blackjack" and "yellow" pines on the basis of growth vigor. The distinction is meaningless botanically but, because of the differences among trees and stands of low and high growth vigor, it is useful in forest management.

This study was coordinated with the continuous forest inventory (CFI) program in the Southwestern Region. A subsample of the 10-point clusters measured in the CFI program was randomly drawn for each Forest. On the Coconino and Tonto National Forests, all of the trees located on the 10-point cluster were felled and measured using procedures similar to those described by Stage and others (1968). Because of a lack of funds, and for safety reasons, not all of the trees were felled on the Lincoln National Forest. Instead, trees were selected to insure representation from all of the diameter classes in the cluster.

EXPECTED MODEL FORMS

Sample tree surface areas were calculated with a modified version of the NETVSL computer program (Stage and others 1968). Before the output from NETVSL was examined, expected model forms were developed. The expected model forms were used to recognize and predict the relationship between surface area and the independent variables of total tree height and d.b.h.

Three simple geometric forms were considered. A tree stem, or portions thereof, can be approximated by a cylinder, cone, or paraboloid. The surface area for each geometric solid can be expressed by the following formulas (Husch 1963):

FORM	FORMULA
Cylinder	$SA = 2\pi rh$
Cone	$SA = \pi r(r^2 + h^2)^{1/2}$
Paraboloid	$SA = [(\pi r)/(6h^2)][(4h^2 + r^2)^{3/2} - r^3]$

where SA = surface area

r = radius of the base, in feet

h = height of the form, in feet

Although the surface area formulas for the cone and the paraboloid appear complex, simplified formulas can be derived without great loss of accuracy. For tree stems, the radius (when expressed in feet) is small compared to height. Therefore, when the height and radius are squared (or cubed) and added (or subtracted) together, the radius contributes little to the result. Hence, the two formulas were simplified:

FORMULA FOR CONE

$$SA = \pi r(r^2 + h^2)^{1/2}$$

$$SA \approx \pi r(h^2)^{1/2}$$

$$SA \approx \pi rh$$

FORMULA FOR PARABOLOID

$$SA = [(\pi r)/6h^2)][(4h^2 + r^2)^{3/2} - r^3]$$

$$SA \approx [(\pi r)/6h^2)][(4h^2)^{3/2}]$$

$$SA \approx 4/3\pi rh$$

From this simplification, it was apparent that the major difference among the surface area formulas was the constant of 2π for cylinders, $4/3\pi$ for paraboloids, and π for cones. The expected model, therefore, is the linear relationship:

$$\text{Surface Area} = b_0 + b_1(DH)$$

where D = diameter at breast height, in inches

H = total tree height, in feet

b_0 = an intercept which should range from 0 to 4.0 square feet. An intercept can be expected because trees 4.5 feet tall or less have $DH = 0$.

b_1 = a constant, which should range from $\frac{4\pi}{(3)(24)}$ to $\frac{\pi}{24}$ (or from 0.1745 to 0.1309) because trees are between a cone and a paraboloid in shape.

ANALYSIS OF UNFORKED TREE DATA

Surface area of unforked trees was plotted over d.b.h. and tree height to determine what samples might be combined. The following tabulation indicates the initial species and National Forest combinations used to develop surface area equations, and the number of observations in each combination. The plotted data also indicated that variance about a regression line would be related to the independent variable, DH .

<i>Species and National Forest</i>	<i>Observations</i>
Douglas-fir on the Lincoln and Tonto National Forests	107
Mexican white pine and white fir on the Lincoln National Forest	77
Blackjack pine on the Coconino National Forest	285
Blackjack pine on the Lincoln and Tonto National Forests	395
Yellow pine on all Forests	165

Least squares regressions of surface area on DH were fitted for the initial species groupings. Observation weights proportional to $1/(DH)^2$ were used to obtain minimum variance estimators of regression coefficients. This initial estimate was based on work with tree volume equations (Cunia 1964), which found that the square residuals increased linearly with the square of the independent variable.

Analysis of covariance (Freese 1964) was used to test for differences between the surface area equations for blackjack pine on the Coconino National Forest and for blackjack pine on the Lincoln and Tonto National Forests. This analysis (table 1) indicated that the differences were not significant at the 5 percent confidence level, so the data from the three Forests were combined and a general equation for blackjack pine was developed. This equation was next tested against the yellow pine equation and the

Table 1.--*Analysis of covariance: comparison of coefficients for unforked blackjack pine equation on the Coconino National Forest and unforked blackjack pine equation on the Lincoln and Tonto National Forests*

Species location and items tested	Degrees of freedom	Residual Sum of squares	Residual mean squares	Calculated F
Coconino	283	2,126.34	7.5136	--
Lincoln and Tonto	393	3,517.91	8.9514	--
Total	676	5,644.25	8.3495	--
Difference for testing slopes	1	2.94	2.94	0.3521 N.S.
Pooled values	677	5,647.19	8.3415	--
Difference for testing levels	1	1.51	1.51	.1813 N.S.
Single equation	678	5,648.70	8.3314	--

Table 2.--*Analysis of covariance: comparison of coefficients for unforked yellow pine equation and unforked blackjack pine equation*

Subspecies and item tested	Degrees of freedom	Residual Sum of squares	Residual mean squares	Calculated F
Yellow pine	163	25,564.70	156.8390	
Blackjack pine	678	5,591.36	8.2468	
Total	841	31,156.06	37.0464	
Difference for testing slopes	1	695.44	695.44	18.772**
Pooled values	842	31,851.50	37.8283	

difference between the equations was significant at the 1 percent confidence level (table 2). However, an equation combining yellow and blackjack pine was also developed for users of these equations who do not differentiate between yellow and blackjack pine. This must be regarded as an equation with an omitted variable--growth vigor. As Kmenta (1971) demonstrated, this could result in biased estimates of the regression coefficients. This situation should not be unduly alarming. The bias involved is a bias of the estimated coefficient as an estimator of the true parameter relating surface area to DH. So long as the equation is used to predict surface area for trees which have the same relationship between DH and the omitted variable, predictions will be unbiased, though inefficient.

Using the final species-Forest groups, the following model was fitted by least squares regression to further examine the form of the residuals:

$$\ln [(\text{Actual SA} - \text{Predicted SA})^2] = c_0 + c_1 \ln (\text{DH})$$

The coefficient c_1 in this equation estimates the power by which the squared residual increases with an increase in DH. The calculated c_1 values are presented in table 3. The powers of DH are close to 2 (the power which was assumed earlier) for all species except yellow pine, which is closer to 1.

Table 3.--*Coefficients for equations of residuals and certain related regression statistics*

Species group	C_0	C_1	Standard error of C_1	Mean square residual (MSR)
Ponderosa pine	-10.78196	2.04002	0.092805	5.00786
Yellow pine	-4.56996	1.19232	.375083	5.13841
Blackjack pine	-9.75613	1.85458	.125818	5.04500
Douglas-fir	-9.77705	1.83869	.220917	3.57956
Mexican white pine, white fir	-9.98315	1.88056	.304233	6.45851

An estimate of the mean squared residual about the regression surface, as conditioned by DH, can be obtained from the equation:

$$\hat{S}_{y \cdot x}^2 = e \left[c_0 + c_1 (DH) + \frac{MSR}{2} \right]$$

where $\frac{MSR}{2}$ = 1/2 mean square residual, which is necessary for conversion of logarithmic estimates to arithmetic units (Baskerville 1972).

A final set of surface area regressions was then fitted with observation weights proportional to $1/(DH)^{C_1}$. These equations and certain regression statistics are presented in table 4. The equation for ponderosa pine was forced through the origin because the intercept was negative, which violated the expected model form. All other coefficients meet expectations.

For those who wish to calculate estimated confidence intervals for predicted values of surface area, the values for mean weighted DH and the weighted corrected sum of squares have been included in table 4.

Table 4.--*Surface area equations and certain regression statistics*

Species group	Surface area equation	$Sy \cdot x$ (Weighted root mean square residual)	Weighted mean DH	Corrected weighted sum of squares
Ponderosa pine	0.146150(DH)	3.2655	242.064	82,142,100
Yellow pine	3.10548 + .149661(DH)	4.8301	1,351.650	83,183,300
Blackjack pine	0.52975 + .143295(DH)	3.0411	248.056	21,302,200
Douglas-fir	3.31473 + .130655(DH)	3.8076	360.927	10,561,300
Mexican white pine, white fir	2.01727 + .137988(DH)	4.6509	298.479	9,370,900

ANALYSIS OF FORKED TREE DATA

Forked tree data were grouped by species, surface areas were plotted over d.b.h. and height, and resulting trends examined. The initial grouping of data and the number of observations for each group were:

<i>Species</i>	<i>Observations</i>
Blackjack pine	62
Yellow pine	15
Douglas-fir, Mexican white pine, white fir	19

The data were next examined by calculating the ratio of the actual forked tree surface area divided by the predicted surface area for a normally formed tree and plotting this ratio over d.b.h., over height of the tallest stem, and over the ratio of height to fork divided by height of the tallest stem. No relationship was apparent over d.b.h. or over height of the tallest stem, but there was a relationship over the ratio of height to fork divided by height of the tallest stem. This relationship appeared to take the form:

$$\left(\frac{FSA}{PSA}\right) = d_0 - d_1 \left(\frac{FH}{TH}\right)$$

where FSA = forked tree surface area, in square feet
 PSA = predicted tree surface area, in square feet, for an unforked tree of the same d.b.h. and height
 FH = height to fork, in feet
 TH = height of tallest stems in the forked tree, in feet

Physical considerations require that as the position of the fork approaches the tip of the tree, the predicted forked tree surface area should approach the prediction for an unforked tree. To meet this requirement, the following model was fitted through the origin for each species group:

$$\left(\frac{FSA}{PSA}\right) - 1 = f\left[1 - \left(\frac{FH}{TH}\right)\right]$$

This model can then be changed back to the original form through these transformations:

$$d_0 = 1 + f$$

$$d_1 = (-f)$$

Analysis of covariance was then used to test differences in coefficients for the three species groups. No significant differences were found at the 5 percent confidence level (table 5), so the data were combined and the following regression was fitted:

$$\left(\frac{FSA}{PSA}\right) = 1.214839 - 0.214839\left(\frac{FH}{TH}\right)$$

To use this equation, the predicted surface area for a normally formed tree is calculated using the equations previously presented in table 4. This predicted surface area is then multiplied by the value calculated in the above equation to estimate the forked tree's surface area. For those users who do not measure the height to the fork, the following equation can be used:

$$\hat{FSA} = 1.086481 (PSA)$$

where \hat{FSA} = estimated forked tree surface area in square feet. This equation is based on the mean value of $\left(\frac{FSA}{PSA}\right)$.

Table 5.--Analysis of covariance: comparison of coefficients for fork and tree equations

Species and items tested	: Degree of : freedom	: Residual : sum of squares	: Residual : mean squares	: Calculated : F
Blackjack pine	60	0.572876	0.0093914	
Yellow pine	13	.301581	.0215415	
Douglas-fir, Mexican white pine, white fir	17	.193569	.0107539	
Total	90	1.068026	0.0118670	
Difference for testing slopes	2	.032780	.0163900	1.3811 N.S.
Pooled values	92	1.100806	.0119653	

LITERATURE CITED

- Baskerville, G. L.
1972. Use of logarithmic regression in the estimation of plant biomass. Can. J. For. 2:49-53.
- Bruce, David.
1970. Predicting product recovery from logs and trees. USDA For. Serv., Res. Pap. PNW-107, 15 p., illus. Pac. Northwest For. and Range Exp. Stn., Portland, Oreg.
- Cunia, T.
1964. Weighted least squares method and construction of volume tables. For. Sci. 10:180-191.
- Davis, Kenneth P., Philip A. Briegleb, John Fedkiw, and L. R. Grosenbaugh.
1962. Determination of allowable annual timber cut on forty-two western National Forests. USDA For. Serv., Washington, D. C. Board Rev. Rep. M-1299, 38 p.
- Freese, Frank.
1964. Linear regression methods for forest research. USDA For. Serv. Res. Pap. FPL-17, 136 p., illus. Forest Products Laboratory, Madison, Wis.
- Grosenbaugh, L. R.
1954. New tree measurement concepts: Height accumulation, giant tree, taper and shape. USDA For. Serv., South. For. and Range Exp. Stn. Occas. Pap. 134, 32 p. New Orleans, La.
- Husch, Bertram.
1963. Forest mensuration and statistics, p. 445-446. The Ronald Press Co., New York.
- Kmenta, J.
1971. Elements of econometrics. 655 p., illus. The MacMillan Co., New York.
- Lexen, Bert.
1943. Bole area as an expression of growing stock. J. For. 41:883-885.
- Stage, Albert R., Richard C. Dodge, and James E. Brickell.
1968. NETVSL--a computer program for calculation of tree volumes with interior defect. USDA For. Serv. Res. Pap. INT-51, 30 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

